Analysis of a Supply Chain Inventory System under Lot Splitting Policy

I-Chen Lin
Department of Industrial Engineering
National Taipei University of Technology

ABSTRACT

The lead times of the vendors are assumed to be Gamma distributions. The lot size quality, measured by the acceptance rate, and the product demand has a normal distribution. The buyer will split his replenishment order to the secondary vendor only if the prescribed service level is achieved and the dual-sourcing total cost should be at least as low as that in the single sourcing scenario.

To analyze the cost economics, We apply MS-EXCEL SOLVER to search for the lowest total cost and determine: (1) the optimal values of (*Q s)* and (2) if the order is split, the upper bound unit price associated with the fraction ordered from the secondary supplier to make the order-splitting a worth-while policy.

Keywords: Inventory, Dual-source, Lead time

分散批量政策下之供應鍊存貨系統分析

林益辰
國立台北科技大學

摘要

供應商的前置時間假設為 Gamma 分配，批量品質用允收率來衡量，其和產品需求同樣假設為常態分配。在預設之服務率可以達成且其總存貨成本比單家供應商採購更低之下，買方將分散一部份之訂購量從第二家供應商採購。考慮其總運送成本，我們建立一個非線性模式，之後應用 MS-EXCEL SOLVER 來求得最低總成本並決定〈1〉最佳訂購量及訂購點〈2〉可接受之第二家供應商所提供之物品單價。

關鍵字：存貨系統、雙供應商、前置時間

1. Introduction

During recent years supply chain management and the supplier (vendor) selection process has drawn considerable attention in the management literature. As a wide range of firms adopt total quality management (TQM) and just-in-time (JIT) concepts, the role of supplier becomes even more important. Although the benefits of a single sourcing system have been widely recognized [4, 24], there are still some inconveniences. As Newman [19] suggests, a single sourcing may lead to loss of technological thrust, excess control, and lack of identity for the supplier.

It is found that although a fraction of U.S. firms have assumed single sourcing from Japanese practice, dual sourcing was their objective and the majority of firms currently practice a supplier base of two or three sources [20]. On the point of diversifying risk, many firms have been suggested that they purchase portions of the order quantity simultaneously from two or more vendors [11]. Early in 1982, Kratz and Cox [15] have documented several case studies where dual sourcing yields a lower unit price and improvements in quality.

In general, a dual sourcing system outperforms a sole sourcing system in safety stocks, delivery performance and therefore, the lower total inventory cost is achieved [12, 23]. We are, in a macro manner, trying to incorporate transportation cost into a non-identical suppliers dual sourcing system with stochastic lead-time under continuous review inventory policy.

1.1. Motivation and Problem Setting

This paper is an attempt to relax the identical suppliers assumption and overcome some of limitations cited before. Thus, we consider a dual sourcing system where two suppliers offer different lot-size qualities, unit prices, transportation rates and also lead performances. One of the suppliers is characterized by better product quality and better lead time performance but higher unit price. In contrast, the other one acts quite the reverse and is willing to markdown its unit price.

In this research, we will explore the dynamics of a supply chain that has the option of using two competing suppliers who offer different unit prices, quality levels, lead-time performances and freight rates. The question we set out to answer in this inventory-logistics framework is what the unit price the latter supplier should offer to make order-splitting a worthwhile policy considering the additional transportation cost and the lot-size quality. If order splitting is attractive, what portion of the order quantity should be split to the latter vendor at the specific price it offered?

The rest of the research is organized in the following way. Section two highlights the relevant literature depicting the dual sourcing systems. Section three introduces an appropriate total cost minimization model using two suppliers with non-identical lead time performances. In section four, the verification and validation of this model are provided by simulation results. Section five presents our conclusions and directions for further studies.

2. Literature Review

Ammer [1] indicates the merits of adopting a multiple sourcing system including reducing the risk of interruption of supply due to quality problems, getting a healthy competition in which each vendor will devote themselves to improving methods and reducing costs in order to get a greater share of the business. And also, the unit price of the item is lower with two or more sources than it would be if all requirements were concentrated on a single source. Kraljic [13] suggests that multiple sourcing should be used for strategic and bottleneck items when there is a high risk of supply, even at a cost premium if necessary. Trevelen [26] also concluded that, in the point of purchasing management, dual sourcing has been touted as an incentive mechanism to motivate suppliers to provide high quality products.

Kratz and Cox's [15] document several case studies where multiple sourcing and its inherent competitive pressures have resulted in lower price as well as quality improvement. Greer et al. [7] assert that competition produces greater savings when vendor firms are operating at low capacity level but the benefits of adopting dual sourcing in reducing procurement costs are not substantial when the industrial is very active. Kratz et al. [14] highlight the main benefit coming from a reduction in unit procurement cost, which leads to overall savings.

Hayya et al. [9] provide the advantages of using two identical vendors by a simulation model. Sculli and Shum [23] propose a mathematical model to derive the formula expressing the mean and variance of the effective lead time. Their study shows the mean and standard deviation of both the effective lead time and the effective lead time demand to be smaller than those of the individual suppliers. Besides, these parameters cited above decrease even more as more suppliers are in use. Their conclusions are much the same as those of Hayya, et al. [9]. Sculli and Wu [22] propose a methodology reducing the number of parameters in the effective lead time density function and indicate that the reorder level required to meet a given no stock-out probability and the buffer stock are both lower than those when only one supplier is used.

Fortuin [6] investigates the impacts of five different lead time distributions (normal, logistic, gamma, lognormal, and weibull) on the total cost. It is found that with respect to both total cost and expected shortage, the type of lead time distribution is not a significant factor, whereas the variance of demand during the lead time is found to be a highly significant factor.

Ramasesh et. al. [21] construct a expected total cost function under the assumptions that the demand per unit time is constant, the lead times of the two vendors are independent and identically distributed random variables (uniform and exponential distributions). Based on the Hooke-Jeeves algorithm, they conclude that when lead times are stochastic, dual sourcing offers savings in inventory holding and backordering costs. Besides, as the variability of the standard deviation of the demand during the lead time increases, the savings will also increase. It is also suggested that dual sourcing is more attractive when the lead time distributions are skewed and long-tailed.

3. Model Development and Solution

One of the main concerns is that we incorporate the freight cost associated with each lot size into the total expected inventory cost function. Most of the studies involved the inventory cost minimization problems only deal with the costs that incur inside his/her firm and set aside the outside costs such as freight cost. In practice, the transportation cost occurs once a replenishment order is placed.

Another consideration is the quality levels of each lot size offered by different vendors. If the buyer performs a receiving screening test for each incoming lot and rejects the defective items, the quantity accepted should be less than the order quantity. Thus, we should use the quantity after screening rather than the order quantity to calculate holding cost and the frequency of placing orders annually.

3.1. Assumptions and Notation

The model assumes that one firm places his/her replenishment order of a single product to two suppliers in the same time. One of the suppliers is characterized by better product quality and better lead time performance but higher unit price. In contrast, the other one acts quite the reverse and is willing to markdown its unit price. To analysis the characteristics of the dual sourcing system with stochastic lead time, demand and acceptance rate, we develop a reorder point, order quantity model with the following assumptions.

No more than one order will be outstanding at any given time. When lead time is stochastic, it is possible for more than one order to be outstanding, although in practice, the probability is low. To ensure no crossover oders, the quantity ordered, Q, will be required to be equal to or greater than the maximal lead time demand. Such an assumption has been widely used in the inventory literature, and found to be effective for finding an optimal order policy [8].

The lead time T_i (i=1,2), independent with the quantity ordered, yields a Gamma distribution with mean $?_{Ti}$ and standard deviation $?_{Ti}$. A gamma distribution is a good fitting of actual data and adopted widely in many inventory problem studies [10]. The demand for this product in unit time (days) is normally distributed with mean $?_D$, standard deviation $?_D$ and also independent of prices, seasons, etc. Thus, the mean lead time demand, $?_L$, can be calculated as $?_T$? $?_D$ [3].

The quality of each lot offered by a vendor is not identical due to the variability of manufacturing processes. The quantity accepted will equal to some percentage of the order quantity. This percentage rate is measured by "acceptance rate", r, which will be also known as a normal distribution with mean $?_r$ and standard deviation $?_r$.

A pre-specified order fill rate, P, is used instead of shortage cost in the model. We should not include the shortage cost if the service level constraint is explicitly included [18]. The measure of service is preferable to other service or shortage-cost criteria in joint transportation-inventory modeling, because it has the capability to calibrate safety stock levels to achieve the same expected annual shortages (service levels) for each transportation option tested [27]. The transportation cost (measured in cents) of each order for supplier i is a function of the amount shipped and is expressed as $\exp(a_i+b_i \ln(Q))$ where $b<0$ [2].

The following summarizes the notation used in this paper. The definitions of some are deferred until they are used.

? (Q, s) = the order quantity and the reorder point for the buyer,

? T_i = the lead times of two suppliers (Gamma distributions with mean $?_{Ti}$ and standard deviation $?_{Ti}$), i = 1, 2,

? R = the amount of expected annual demand for this product,

? A = the cost of placing one or more orders,

? O = the cost of receiving one incoming order,

? v = the unit value or price,

? i = the holding cost factor (%),

? $?_L$ = the mean lead time demand and equals to $?_{Ti}$? $?_D$,

? a , b = the estimated transportation rates.

3.2. The Total Expected Inventory Cost

The total annual inventory-logistics cost can be expressed as the summation of ordering cost, in-transit stock holding cost, cycle-stock holding cost, safety stock carrying cost, transportation cost and purchasing cost.

If Q? is the total quantity ordered and f is the fraction that is ordered to the supplier with poorer acceptance rate, $?_{r2}$, and lower unit price v_2. Therefore, when the reorder point is reached, the buyer places the order of size $Q(1-f)$ to another supplier with unit price v_1 and accepted rate $?_{r1}$. After receiving screen inspection, the replenished quantity available can be calculated as

$$Q' ? ?_{r1}Q(1 ? f) ? ?_{r2}Qf.$$

The unit value v_c in the dual sourcing scenario compound v_1, v_2 and should be weighted by $(1-f)$ and f respectively. It can be expressed as

$$v_c ? (1 ? f)v_1 ? fv_2.$$

There is Q' accepted in a replenishment cycle, and (R/Q') orders per year with each costing $(A+2O)$. Since, there will be two receiving inspection costs result from different suppliers in each ordering cycle. The total ordering cost will be

$$(\frac{R}{Q'})(A ? 2O).$$

Since T_1 and T_2 are stochastic, supplier-2's order may arrive before or after supplier-1s' regardless of whether $?_{T2}$ exceeds $?_{T1}$. Therefore, two separate situations must be considered [17]: the situation where supplier-1's order arrives before supplier-2's; i.e., $T_2 > T_1$ and the situation where supplier-2's order arrives before supplier-1's, i.e., $T_1 > T_2$. According to Lau's work [16], the average inventory level can be expressed as

$$\begin{aligned} I &? \frac{Q'}{2} ? f[E(T_2) ? E(T_1)]?_D ? s ? E(T_1)?_D \\ &? \frac{Q'}{2} ? f(?_{T_2} ? ?_{T_1})?_D ? s ? ?_{T_1}?_D. \end{aligned}$$

Thus, the holding cost can be calculated as the average stock level times weighted value times the holding cost factor.

$$\left(\frac{Q'}{2} ? (?_{T_2} ? ?_{T_1})?_D f ? s ? ?_{T_1}?_D\right)v_c i.$$

We use the exponential function, $\exp(a + b\ln(Q))$ [2], to estimate the transportation rate as a function of the order quantity Q. The annual transportation cost can be calculate as the number of orders, times the quantity ordered, times the freight rate (cent/unit) for each lot-size. In dual sourcing system, it will be

$$\left(\begin{array}{l} Q(1 ? f)\exp(a_1 ? b_1\ln(Q(1 ? f))) \\ ? Qf\exp(a_2 ? b_2\ln(fQ)) \end{array}\right)\frac{R}{100Q'}.$$

Each unit shipped will be in transit for an average of $?_T$ period. Since the average demand per period is $?_D$, an average of $?_T?_D$ units will be in transit. By multiplying this average unit by its unit value the in-transit cost is

$$(1 ? f)?_D?_{T1}v_1 i ? f?_D?_{T2}v_2 i.$$

Thus, we can formulate the total expected cost function as the summation of above cost function and the purchasing cost:

$$ETAC(Q, s) ? (\frac{R}{Q'})(A ? 2O) ?$$

$$? \frac{Q'}{2} ? (?_{T2} ? ?_{T1})?_D f ? s ? ?_{T1}?_D ? v_c i ?$$

$$? \begin{matrix} Q(1 ? f)\exp(a_1 ? b_1 \ln(Q(1 ? f))) \\ ? Qf \exp(a_2 ? b_2 \ln(fQ)) \end{matrix} ? \frac{R}{100Q'} ?$$

$$?(1 ? f)?_D ?_{T1} v_1 i ? f?_D ?_{T2} v_2 i ?? Rv_c.$$

3.3. Estimating the Expected Shortage

If the unit demand follows a normal distribution with mean $?_D$ and standard deviation $?_D$, the conditional distribution of demand given the lead time $T_l = j, j = 1\ldots n$ is also a normal distribution with mean $j?_D$ and standard deviation $?j?_D$ [25]. Thus, one can calculate the conditional expected shortage given lead time $T_l = j_j$ as

$$\mathrm{ESSR}_j ? G_u(k_j)?_j,$$

$$G_u(k_j) ? \frac{1}{2?} ?_{k_j}^{?} (u ? k_j)\exp(\frac{? u^2}{2})du,$$

$$\text{where } k_j ? u(s) ? \frac{(s ? u_j)}{?_j},\ u_j ? j?_D,\ ?_j ? \sqrt{j}?_D.$$

The function can be calculated as

$$f(k_j) ? k_j(1 ? F(k_j)),$$

where $f(k_j)$ and $F(k_j)$ are the probability density function and the cumulative density function of the standard normal distribution respectively.

If T_i is the lead time distributions for supplier-i where i =1, 2, the first shipment arrival time distribution can therefore be determined as

$$T_{[1]} ? \min ? T_1, T_2 ?$$

The expected shortages per replenishment cycle in the dual sourcing system now can be determined as

$$ESSR ? \sum_{j?1}^{n} ESSR_j P_j^{?1?},$$

where $P_j^{[1]}$ represents the probability of $T_{[1]} = j$. Let $F_{[1]}(t)$ denote the cumulative distribution function of $T_{[1]}$. $P_j^{[1]}$ can be computed using the following formula:

$$P_j^{[1]} ? F_{[1]}(j) - F_{[1]}(j-1), \quad j ? 1 ? n,$$

where

$F_{[1]}(t) = P(T_{[1]} ? \ t) = 1- P(T_{[1]} > t) = 1- P(T_1 > t, T_2 > t) = 1- (1 - F_1(t))(1 - F_2(t))$.

On the other hand, once the order fill rate is set, we can define the target number of units short per replenishment cycle, TS as $(1-P)(1-f)?_{r1}Q$, where $(1-P)$ represents the probability that the demand exceeds the order quantity in a cycle. The average inventory quantity accepted in each replenishment order is $(1-f)?_{r1}Q$. It should be satisfied that the expected shortage is less than or equal to the target number of shortage, TS.

Therefore, the problem is to find Q and s that minimize the total cost function subject to

$$ESSR ? (1-P)(1-f)?_{r1}Q,$$

$$Q' ? ?_{r1}Q(1? f) ? ?_{r2}Qf,$$

$$Q ? 1,$$

$$s ? 0.$$

We thus, build this model in MS-EXCEL and search the optimal value of Q and s by SOLVER.

4. Model Verification

The model in Section 3 is based on a number of simplifying assumptions. This section will exhibit how a simulation system, based on BASIC, was used to verify the approximate model. We prefer SIMPAK, a random variable generator developed by Donaghey [5] to any commercial simulation software.

The purpose of the simulation was to obtain the minimal total cost by setting initial values Q and s. This value will be compare to the actual values to arrive at the accuracy of the model. To account for the initialization accuracy, several pilot runs were run for 15 years to collect the simulation statistics to make sure that this length was enough to eliminate the bias and to provide independence between successive replications of a simulation run. After verifying that the simulation model was providing useful measures of system performance, we used the output to test the accuracy of the analytical model.

The measure of model accuracy, Δ is measured as the difference between the mean point estimate of the total cost from the simulation and the mode's prediction. The difference of mean total expected cost (Q =1899, s =188) between the simulation result ($30,651.9) and model's prediction ($30.678.38) is only 0.086% of the prediction value. It shows a good fitting of the model.

1. Conclusion and Directions for Future Research

In dual sourcing, the unit price of the secondary vendor can be determined by both lot size acceptance rate and the ratio of split order. For a given value of *f*, the unit price is determined by the lot size acceptance rate such that the split order policy is attractive to the buyer. On the other hand, the secondary vendor has to offer price discount to make the buyer spilt his/her order economically or, says make the dual sourcing total cost as low as it in single sourcing. Figure 1 illustrates the relationships between the split order ratio and the unit price offered by the secondary vendor. As we expected, the more unreliable the lot size quality is, the more price discount the vendor should offer. With the split order ratio *f* increases, the dual sourcing total expected cost increases and the threshold of price discount offered by the secondary vendor becomes lower.

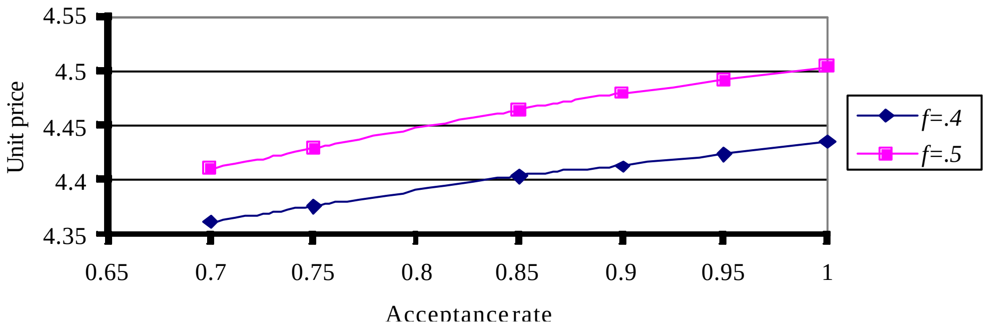

Figure1 The relationships between price and acceptance rate of the secondary vendor at f=.4

We have investigated the supply chain decision-making that is involved in a dual sourcing system. The primary supplier is characterized as better lead time performance, more reliable lot size quality, low freight rate but higher unit price. The secondary supplier is attractive because of its pricing flexibility. We modeled the situation in a broader inventory-logistics aspect that covered the in-transit inventories, transportation cost and the lot size quality. In this framework, it is suggested that dual sourcing or order splitting may not be favorable if the secondary vendor can't offer a price discount surpassing the threshold we obtained. We provided a simple method and sample exchange curves to determine the fraction of the order placed with the secondary vendor along with the unit price threshold for the vendor to make the order splitting a worth-while policy.

The following are some suggestions for future research:

The research does not consider the coordination of more than two suppliers, thought many buyers do use more than two suppliers for a given item. Extensions to deal with more than two suppliers and determine the decision rules for allocating the order quantity associated with each vendor may be considered.

With the global supply chain prevails, planning and control in multi-echelon stocking problems becomes more and more important. In a single-echelon system, we assume reasonably smooth demand whose residual variability can be predicted by statistical procedures and independent to any other variable. Furthermore, we assume that the inventory should be replenished as soon as its level drops below a prescribed value (reorder point) or on a regular basis (periodic review). These necessary assumptions in single-echelon are normally not valid in a multi-echelon system. How to model these related problems should draw more attentions of researchers.

References

[1] Ammer, D. S., *Material Management and Purchasing*, Richard D. Irwin, Inc., IL. 1980.

[2] Arcelus, F. J. and Rowcroft, J. E. (1991), Small Order Transportation Costs in Inventory Control, *Logistics and Transportation Review*, Vol. 27, No. 1, pp.3-13.

[3] Clark, C. E. (1961), The Greatest of a Finite Set of Random Variables, *Operations Research* , 9, pp.145-162.

[4] Davis, M. M., Aquilano, N. J., Chase, R. B., *Fundamentals of Operations Management*, Irwin McGraw-Hill, 3rd ed. 1999.

[5] Donaghey, C. E., *Simpak User's Manua*l, Dept. of I.E., University of Houston, TX. 1994.

[6] Fortuin, L. (1980), Five Popular Probability Density Functions: A Comparision in the Field of Stock Control Models, *Journal of the Operations Research Society*, Vol. 31, pp.937-942.

[7] Greer, W. R., and Liao, S. S, *Cost Analysis for Dual Source: Weapons Procurement*, Naval Postgraduate School, Monterey, CA. 1983.

[8] Hadley, G. and Whitin, T., *Analysis of Inventory System*, Prentice Hall, Upper Saddle River, N. J. 1963.

[9] Hayya, J. C., Christy, D. P. and Pan, A. (1987), Reducing Inventory Uncertainty: A Reorder Point System with Two Vendors, *Production Inventory Management*, 28, pp.43-49.

[10] Hayya, J. C., Ord, J. K. and Pan, A. (1986), On Lot-splitting with Two Vendors: the Deterministic (s, Q) Inventory System with and without Planned Shortages, *DSI Proceedings*, pp.1103-1105.

[11] Horowitz, I. (1986), On Two-source Factor Purchasing, *Decision Science*, 17, pp.274-279.

[12] Kelle, P., Silver, E. A. (1990), Decreasing Expected Shortages through Order Splitting, *Costs and Production Economics*, 19, pp.351-357.

[13] Kraljic, P., Purchasing Must Become Material Management, *Harvard Business Review*, Sept.-Oct., 1983.

[14] Kratz, D. and Hiller, C., *Establishing Competitive Production Services: A Handbook for Program Managers*, Defense Systems Management College, Fairfax, VA., 1984.

[15] Kratz, L. A. and Cox, L., Competition of Defense Procurements: Evidence, Theory and Application, *Federal Acquisition Research Symposium*, Washington, DC., 1982

[16] Lau, H. and Zhao, L. (1993), Optimal Ordering Policies with Two Suppliers When Lead time and Demand Are All Stochastic, *European Journal of Operational Research* , 68, pp.120-133.

[17] Lau, H. S. and Lau, H. (1994), Coordinating Two Suppliers with Offsetting Lead Time and Price Performance, *Journal of Operations Management*, Vol. 11, pp.327-337.

[18] Moon, I. and Choi, S. (1998), A Note on Lead Time and Distributional Assumptions in Continuous Review Inventory Models, *Computers & Operations Research*, Vol. 25, No. 11, pp.1007-1012.

[19] Newman, R.G. (1988), Single Sourcing: Short Term Saving versus Long Term Problem, *Journal of Purchasing and Materials Management*, 25(2), pp.20-25.

[20] Presutti, W. D. (1992), The Single Sourcing Issue: US and Japanese Sourcing Strategies, *International Journal of Purchasing and Materials Management*, pp.2-9.

[21] Ramasesh, R. V., Ord, J. K., Hayya, J. C. and Pan, A. (1991), Sole versus Dual Sourcing in Stochastic Lead Time (s, Q) Inventory Models, *Management Science*, 37, pp.428-443.

[22] Sculli, D. and Wu, S. Y. (1981), Stock Control with Two Suppliers and Normal Lead Time, *Journal of the Operational Research Society*, 32, pp.1003-1009.

[23] Sculli, D., Shum, Y. W. (1990), Analysis of a Continuous Review Stock Control Model with Multiple Suppliers, *Journal of the Operational Research Society*, 41(9), pp.873-877.

[24] Sheridan, J. H. (1988), Strategic Manufacturing: Betting on a Single Source, *Industrial Week* , 236(3), pp.31-36.

[25] Silver, E. A. and Peterson, R., *Decision Systems for Inventory Management and Production Planning*, Wiley, N. J., 1985.

[26] Trevelen, M. (1987), Single Sourcing: A Management for the Quality Supplier, *Journal of Purchasing and Materials Management*, April, pp.49-55.

[27] Tyworth, J. E. (1992), Modeling Transportation-Inventory Trade-Offs in a StochasticSetting, *Journal of Business Logistics*, Vol. 13(2), pp.97-124.